BOOLEAN ALGEBRA

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1 MARK QUESTIONS

1. Write the Sum of Product form of the function *F (P,Q,R)* for the following truth table representation of F:

P	*Q*	*R*	*F*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

1. Sol

P	***Q***	***R***	***F***	**Minterm**
0	0	0	1	P’Q’R’
0	0	1	0	P’Q’R
0	1	0	0	P’Q R’
0	1	1	1	P’Q R
1	0	0	0	P Q’R’
1	0	1	0	P Q’R
1	1	0	1	P O R’
1	1	1	1	P Q R

Sum of Product form of function
F(P, Q, R) is
F(P,Q, R) = P’Q’R’+ P’Q R + PQ R’+ P Q R

2. Write the Product of Sum form of the function *F(X,Y,Z)* for the following truth table representation of F:

X	*Y*	*Z*	*F*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

SOL

X	***Y***	***Z***	***F***	**Maxterm**
0	0	0	1	X + Y + Z
0	0	1	0	X + Y + Z’
0	1	0	0	X + Y’+ Z
0	1	1	1	X + Y’+ Z’
1	0	0	0	X’ + Y + Z

1	0	1	0	X' + Y + Z'
1	1	0	1	X' + Y'+ Z
1	1	1	1	X' + Y'+ Z'

So, *F* = (X + Y + Z'). (X + Y' + Z).
(X' + Y + Z). (X' + Y + Z').

3. Write the Product of Sum of the function *G(U,V,W)* for the following truth table representation of G:

U	*V*	*W*	*G*
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SOL

U	***V***	***W***	***G***	**Maxterm**
0	0	0	1	U + V +W
0	0	1	0	U + V + W'
0	1	0	1	U + V'+ W
0	1	1	0	U + V'+ W'
1	0	0	1	U' + V + W
1	0	1	0	U' + V+ W'
1	1	0	0	U' + V'+ W
1	1	1	1	U' + V'+ W'

To get the Product of Sum form, we need to product maxterms for all those input combinations that product output as 0. Thus, G(U, V, W) = (U + V + W'). (U + V' + W').
(U' + V + W'). (U' + V' + W)

4. Write the Product of Sum form of the function *G (U, V, W*) for the following truth table representation of G:

U	*V*	*W*	*G(U, V, W)*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

SOL

U	*V*	*W*	*G*	**Maxterm**
0	0	0	0	U + V +W
0	0	1	1	U + V + W’
0	1	0	0	U + V’+ W
0	1	1	1	U + V’+ W’
1	0	0	1	U’ + V + W
1	0	1	0	U’ + V+ W’
1	1	0	1	U’ + V’+ W
1	1	1	0	U’ + V’+ W’

To get the Product of Sum (POS) form, we need to product maxterms for all those input combinations that produce output as 0. Thus,

$$G(U, V, W) = (U + V + W).(U + \overline{V} + W).$$
$$(\overline{U} + V + \overline{W}).\ (\overline{U} + \overline{V} + \overline{W})$$

5. Write the Sum of Product form of the function F(A, B, C) for the following truth table representation of F:

A	*B*	*C*	*F*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SOL

A	***B***	***C***	***F***	**Minterm**
0	0	0	0	A’BC’
0	0	1	0	A’B’C
0	1	0	1	A’BC’
0	1	1	1	A’BC
1	0	0	1	AB’C’
1	0	1	0	AB’C
1	1	0	0	ABC’
1	1	1	1	ABC

To get the SOP form, we need to sum minterms for all those input combinations that produce output
as 1. Thus,
F(A,B,C) = A’BC’+A’BC + AB’C+ABC

6. Write the POS form of boolean function G, which is represented in a truth table as follow*s:*

A	*B*	*C*	*G*
0	0	0	0

0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

SOL

A	***B***	***C***	***G***	**Maxterm**
0	0	0	0	$A + B + C$
0	0	1	1	$A + B + \overline{C}$
0	1	0	1	$A + \overline{B} + C$
0	1	1	0	$A + \overline{B} + \overline{C}$
1	0	0	0	$\overline{A} + B + C$
1	0	1	0	$\overline{A} + B + \overline{C}$
1	1	0	1	$\overline{A} + \overline{B} + C$
1	1	1	1	$\overline{A} + \overline{B} + \overline{C}$

To get the Product of Sum (POS) form, we need to product maxterms for all those input combinations that produce output as 0. Thus,

$G (A, B, C) = (A + B + C). (A + \overline{B} + \overline{C})$
$.(\overline{A} + B + C). (\overline{A} + B + \overline{C})$

7. Write the SOP form of Boolean function *F*, which is represented in a truth table as follows:

X	*Y*	*Z*	*F*
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SOL

X	***Y***	***Z***	***F***	**Minterm**	
	0	0	0	1	$\overline{X} + \overline{Y} + \overline{Z}$
	0	0	1	0	$\overline{X} + \overline{Y} + Z$
	0	1	0	1	$\overline{X} + Y + \overline{Z}$
	0	1	1	0	$\overline{X} + Y + Z$
	1	0	0	1	$X + \overline{Y} + \overline{Z}$
	1	0	1	0	$X + \overline{Y} + Z$
	1	1	0	0	$X + Y + \overline{Z}$
	1	1	1	1	$X + Y + Z$

To get the SOP form, we need to sum minterms for all those input combinations that produce outputs as 1. Thus,

$$F\ (X, Y, Z) = (\overline{X}.\ \overline{Y}.\ \overline{Z}) + (\overline{X}.\ Y.\ \overline{Z}) + (X.\ \overline{Y}\ \overline{Z}) + (X.\ Y.\ Z)$$

8. Write the POS form of a boolean function *F,* which is represented in truth table as follows:

A	*B*	*C*	*F*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SOL

A	***B***	***C***	***F***	**Maxterm**
0	0	0	0	$A + B + C$
0	0	1	1	$A + B + \overline{C}$
0	1	0	1	$A + \overline{B} + C$
0	1	1	0	$A + B + C$
1	0	0	1	$\overline{A} + B + C$
1	0	1	0	$\overline{A} + B + \overline{C}$
1	1	0	0	$\overline{A} + \overline{B} + C$
1	1	1	1	$\overline{A} + \overline{B} + \overline{C}$

To get the POS form, we need to product maxterms for all those input combinations that produce output as 0. Thus,

$$F(A, B, C) = (A + B + C).\ (A + \overline{B} + \overline{C})\ .(\overline{A} + B + \overline{C}).\ (\overline{A} + \overline{B} + C)$$

9. Write the SOP form of a boolean function F, which is represented in a truth table as follows:

A	*B*	*C*	*F*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

SOL

A	***B***	***C***	***F***	**Minterm**
0	0	0	1	$\overline{A}\ .\overline{B}\ .\overline{C}$
0	0	1	0	$\overline{A}\ .B\ .C$

0	1	0	0	$\bar{A}.B.C$
0	1	1	1	$\bar{A}.B.C$
1	0	0	0	$A.\bar{B}.\bar{C}$
1	0	1	0	$A.\bar{B}.C$
1	1	0	1	$A.B.\bar{C}$
1	1	1	1	$A.B.C$

To get the SOP form, we need to sum minterms for all those input combinations that produce output as 1. Thus,

$$F(A, B, C) = (\bar{A} . \bar{B} . \bar{C}) + (\bar{A} . B. C) + (A .B . \bar{C}) + (A . B . C)$$

10. Write the SOP form of a boolean function *F*, which is represented in a truth table as follows:

X	*Y*	*Z*	*F(X, Y, Z)*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

X	***Y***	***Z***	***F(X, Y, Z)***	**Minterm**
0	0	0	1	$\bar{X} + \bar{Y} + \bar{Z}$
0	0	1	1	$\bar{X} + \bar{Y} + Z$
0	1	0	0	$\bar{X} + Y + \bar{Z}$
0	1	1	1	$\bar{X} + Y + Z$
1	0	0	1	$X + \bar{Y} + \bar{Z}$
1	0	1	0	$X + \bar{Y} + Z$
1	1	0	0	$X + Y + \bar{Z}$
1	1	1	1	$X + Y + Z$

To get the SOP form, we need to sum minterms for all those input combinations that produce output as 1. Thus,

$$F(X, Y, Z) = (\bar{X}. \bar{Y}. \bar{Z}) + (X. \bar{Y}. Z) + (\bar{X}. Y. Z) + (X. \bar{Y} \bar{Z}) + (X. Y. Z)$$

11. Write the POS form of boolean function H, which is represented in a truth table as follows:

X	*Y*	*Z*	*H*
0	0	0	1
0	0	1	0
0	1	0	1

0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SOL

X	***Y***	***Z***	***H***	**Maxterm**
0	0	0	1	X + Y + Z
0	0	1	0	X + Y + Z
0	1	0	1	X + Y+ Z
0	1	1	1	X + Y + Z
1	0	0	1	X + Y + Z
1	0	1	0	X + Y+ Z
1	1	0	0	X + Y+ Z
1 1	1	1	X + Y+ Z	

To get the POS form, we need to maxterms for all those input combinations that produce output as 0.Thus,

H (X, Y, Z) = $(X + Y + \overline{Z}) . (\overline{X} + Y + \overline{Z}) . (\overline{X} + \overline{Y} + Z)$

12. Write the SOP form of boolean function G, which is represented in truth table as follows:

P	*Q*	*R*	*G*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

SOL

P	***Q***	***R***	***G***	**Minterm**
0	0	0	0	$\overline{P} . \overline{Q} . \overline{R}$
0	0	1	0	$\overline{P} . \overline{Q} . R$
0	1	0	1	$\overline{P} . Q . \overline{R}$
0	1	1	1	$\overline{P} . Q . R$
1	0	0	1	$P . \overline{Q} . \overline{R}$
1	0	1	0	$P . \overline{Q} . R$
1	1	0	1	$P . Q . \overline{R}$
1	1	1	1	P . Q . R

To get the SOP form, we need to sum minterms for all those combinations that produce output as 1. Thus,

G (P, Q, R) = $(\overline{P} . Q . \overline{R}) + (\overline{P} . Q . R) + (P . \overline{Q} . \overline{R}) + (P . Q . \overline{R}) + (P . Q . R)$

13. Write the POS form of boolean function *H*, which is represented in a truth table as follows:

A	*B*	*C*	*H*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SOL

A	***B***	***C***	***H***	**Maxterm**
0	0	0	0	$A + B + C$
0	0	1	1	$A + B + \overline{C}$
0	1	0	1	$A + \overline{B} + C$
0	1	1	1	$A + \overline{B} + \overline{C}$
1	0	0	1	$\overline{A} + B + C$
1	0	1	0	$\overline{A} + B + \overline{C}$
1	1	0	0	$\overline{A} + \overline{B} + C$
1	1	1	1	$\overline{A} + \overline{B} + \overline{C}$

To get the POS from, we need to product maxterms for all those input combinations that produce output as 0. Thus,

$$H(A, B, C) = (A + B + C).\ (\overline{A} + B + \overline{C})\ .\ (\overline{A} + \overline{B} + C).$$

14. Write the POS form a boolean function G, which is represented in a truth table as follows:

u	*v*	*w*	G
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

SOL

u	***v***	***w***	***G***	**Maxterm**
0	0	0	1	$u + v + w$
0	0	1	1	$u + v + \overline{w}$
0	1	0	0	$u + \overline{v} + w$
0	1	1	0	$u + \overline{v} + \overline{w}$

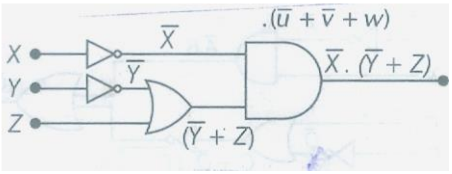
1	0	0	1	$\overline{u} + v + w$
1	0	1	1	$\overline{u} + v + \overline{w}$
1	1	0	0	$\overline{u} + \overline{v} + w$
1	1	1	1	$\overline{u} + \overline{v} + \overline{w}$

To get the POS form, we need to product maxterms for all those input combinations that produce output as 0. Thus,

$$G(u, v, w)= (u + \overline{v} + w).(u + \overline{v} + \overline{w}).\ (\overline{u} + \overline{v} + w)\ ) .(\overline{u} + \overline{v} + w)$$

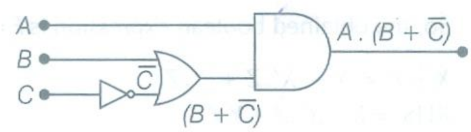
15. Draw a logic circuit diagram for the boolean expression: $\overline{X}.(\overline{Y} + Z)$

SOL



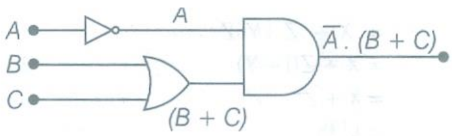
16. Draw a logic circuit diagram for the boolean expression: $A.(B+\overline{C})$

SOL



17. Draw a logic circuit diagram for the boolean expression: $\overline{A}.(B+C)$

SOL



18. Prove that X . (X + Y) = X by truth table method.

SOL

X	Y	X + Y	X. (X + Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

From the above table it is obvious that X . (X + Y) = X because both the columns are identical.

19. Find the complement of the following Boolean function:

$F_1 = AB' C'D'$

SOL

(AB' +CD') = (AB')' . (C'D')' (De Morgan's first theorem)

= (A' + B'') . (C'' + D'') (De Morgan's second theorem

i.e. A.B = A + B)

= (A' + B) .(C + D)

(X'' = X)

20. In the Boolean Algebra, verify using truth table that X + XY for each X, y in (0, 1).

SOL

As the expression X + XY is a two variable expression, so we require possible combinations Of values of X, Y. Truth Table will be as follows:

X	Y	X + Y	X. (X + Y)
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Comparing the columns X + XY and X, we find, contents of both the columns are identical, hence verified.

21. In the Boolean Algebra, verify using truth table that (X + Y)' + X'Y' for each X' Y in (0, 1).

Sol

As it is a 2 variable expression, truth table will be as follows:

X	Y	X + Y	(X + Y)'	X'	Y'	X'Y'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Comparing the columns (X + Y)' and X'Y', both of the columns are identical, hence verified.

22. Give the dual of the following result in Boolean Algebra

X . X' = for each X.

Sol

Using duality principle, dual of X . X' = 0 is X + X' = 1 (By changing (.) to (+) and viceversa and by replacing 1's by 0's and vice versa).

23. Define the followings:

(a) Minterm (b) Maxterm (c) Canonical form

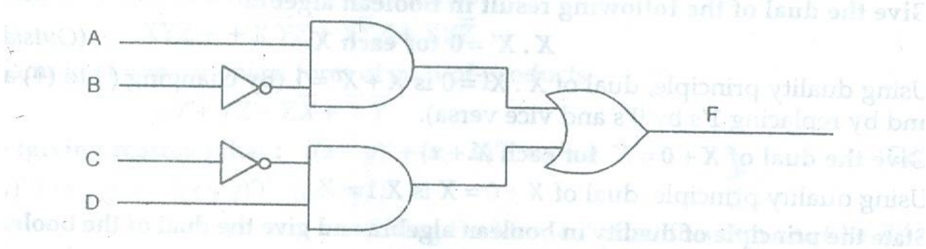
Sol

(a) A Minterm is a product of all the literals (with or without the bar) within the logic system.

(b) A Maxterm is a sum of all the literals (with or without the bar) within the logic system.

(c) A boolean expression composed entirely either of minterms or Maxterms is referred to canonical expression.

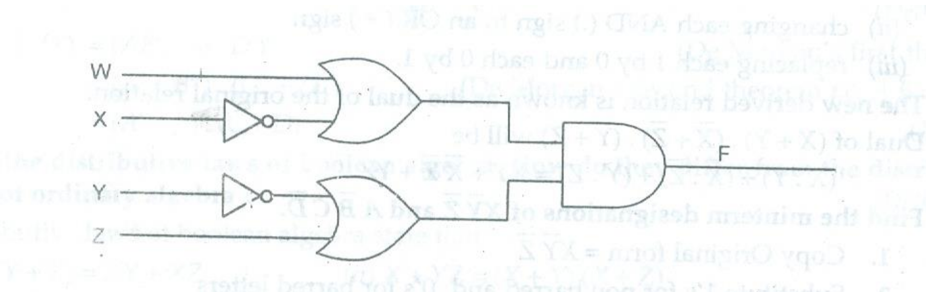
24. Interpret the following logic Circuit as Boolean expression:



Sol

F = AB + CD

25. Interpret the following Logic Circuit as Boolean Expression:



Sol

F =(W + X) (Y + Z).

26. Write the dual of the Boolean expression A + B’ .C.

Sol

Dual of the Boolean expression *A + B’ .C is A . (B’ + C).*

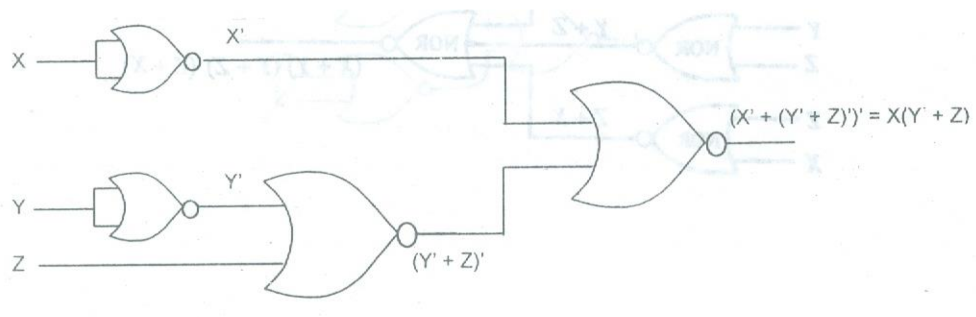
27. Write the dual of the Boolean expression (B’ + C) . A.

Sol

Dual of the Boolean expression *(B’ + C) . A is (B’ .C) + A.*

28. Represent the boolean expression X (Y’ + Z) with help of NOR gates only.

Sol



29. State Demorgan’s Laws:

Sol

De Morgan’s first theorem. It states that X + Y = X . Y

De Morgan’s second theorem. It states that X . Y = X + Y

30. Which gates are called Universal gates and why?

Sol

NAND and NOR gates are less expensive and easier to design. Also, other switching functions and (AND, OR) can easily be implemented using NAND/NOR gates. Thus, these (NAND/NOR) gates are also referred to *as Universal Gates.*

2 Marks Questions

1. Name the law shown below and verify it using a truth table.

A + B. C = (A + B) .(A + C)

Sol

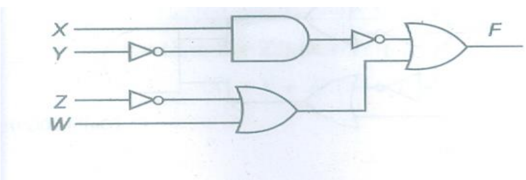
A + B.C =(A + B).(A +C)

The above stated law is called distributive law.

A	B	C	B.C	A+B.C	A+B	A+C	(A+B).(A+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Since, the column 5 and column 8 are equal. Hence, the given law A+B.C = (A + B).(A + C) is verified.

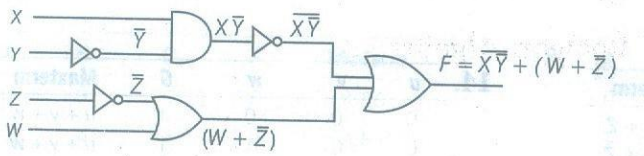
2. Obtain the Boolean expression for the logic circuit shown below:



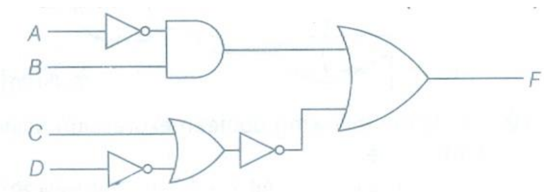
Name the law shown below and verify it using a truth table.

X + X’. Y = X + Y

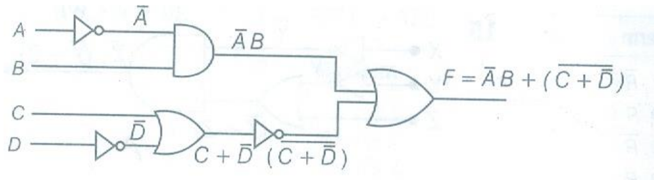
Sol



3 Obtain the Boolean expression for the logic circuit shown below:



Sol



So, the obtained boolean expression is $F = \overline{A}B + (\overline{C + \overline{D}})$

4 Verify the following using boolean laws

X + Z = X + X’.Z + Y.Z

Sol

X + Z = X + X’.Z + Y. Z

RHS = X + X’ .Z + Y.Z

= (X + X’). (X + Z) + Y. Z (by using distributive law)

= 1.(X + Z) + Y.Z (by using X + X’ = 1)

= X + Z + Y. Z

= X + Z(1+Y)

= X + Z (by using absorption law)

= LHS **Hence proved**

5 Verify the following using boolean laws

A + C = A + A’.C + B.C

Sol

A + C = A + A’.C + B.C

RHS = A + A’ .C + B.C

= (A + A’). (A + C) + B.C (by using distributive law)

= 1.(A + C) + B.C (by using A + A’ = 1)

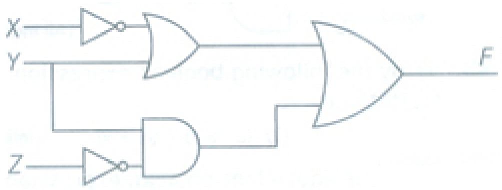
= A + C + B. C

= A + C. (1+B)

= A + C (by using absorption law)

= LHS **Hence proved**

6 Obtain the Boolean expression for the logic circuit shown below:



Sol
So, the obtained boolean expression is *F* = (X' + Y) + Y. Z'

7 State DeMorgan's laws. Verify one of the DeMorgan's laws using a truth table.
Sol
DeMorgan's Laws:

It states that

(i) $\overline{(A + B)} = \overline{A}.\ \overline{B}$ (ii) $\overline{(A\ .\ B)} = \overline{A} + \overline{B}$

Truth table for $\overline{A + B} = \overline{A}\ .\overline{B}$

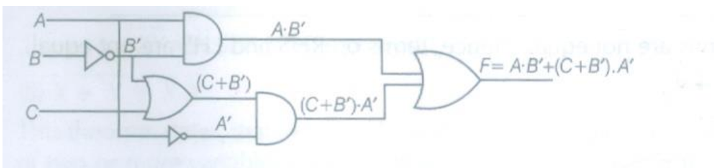
A	**B**	**A+B**	$\overline{\textbf{(A+B)}}$	$\overline{\textbf{A}}$	$\overline{\textbf{B}}$	$\overline{\textbf{A}}.\overline{\textbf{B}}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Column 4 and Column 7 are equal, first law is proved.

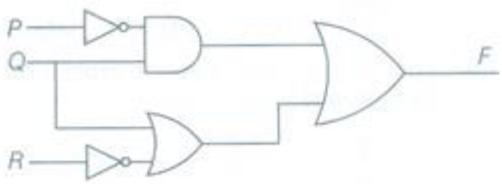
8 Draw a logic circuit for the following boolean expression.

A.B' +(C +B') .A'

Sol
F = A . B' + (C + B') . A'



9 Obtain the Boolean expression for the logic circuit shown below:



Sol
So, the obtained boolean expression is *F* = P'Q + (Q + R')

10 Verify the following using boolean expression using truth table:

(i) X + 0 = X

(ii) X + X' = 1

Sol

(i) X + 0 = X

X	0	X + 0
0	0	0
1	0	1

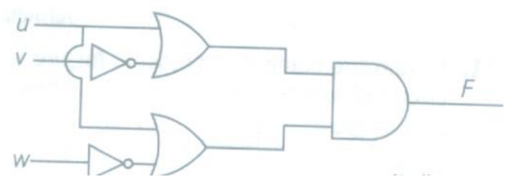
So, X + 0 + X

(ii) X + X' = 1

X	X'	X + X'
0	1	1
1	0	1

As X + X' = 1. Hence proved.

11 Write the equivalent Boolean expression for the following logic circuit:



Sol

So, the obtained boolean expression if *F* = (u + $\overline{v}$) .(u + $\overline{w}$)

12 Verify the following boolean expression using truth table:

(i) X . X' = 0 (ii) X + 1 = 1

Sol

(i) X.X' = 0

X	X'	X.X'
0	1	0
1	0	0

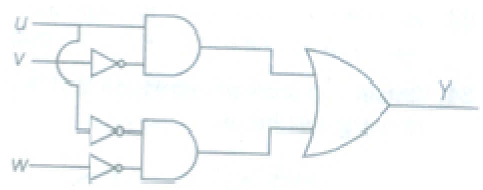
As X. X' = 0. Hence proved.

(ii) X + 1 = 1

X	1	X + 1
0	1	1
1	1	1

As X + 1 = 1. Hence proved.

13 Write the equivalent boolean expression for the following logic circuit:



Sol

So, the obtained boolean expression is Y = (u, v') + (u' . w')

14 Verify the following boolean expression using truth table:

u. (u' + v) = (u + v)

sol

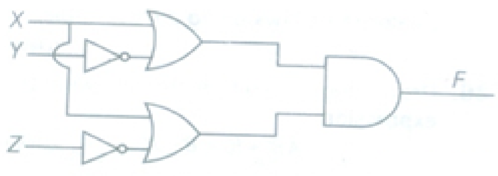
u .(u' + v) = u + v

u	v	u'	u'+ v	u + v	u.(u' + v)
0	0	1	1	0	0
0	1	1	1	1	0
1	0	0	0	1	0
1	1	0	1	1	1

As u .(u' + v) and u + v columns are not equal. Hence, terms on RHS and LHS are not equal.

Hence, u .(u' + v) # u + v

15. Write the equivalent boolean expression for the following logic circuit:



Sol

So, the obtained boolean expression is *F* = (X + Y'). (X + Z')

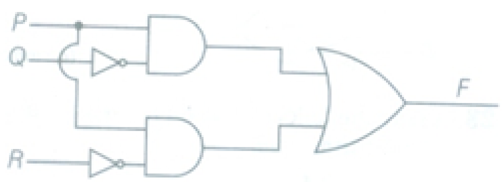
16. Verify the following boolean expression using truth table:

X + Y .Z = (X + Y) . (X +Z)

Sol

X	Y	Z	YZ	X + YZ	X + Y	X + Z	(X + Y).(X + Z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

17. Write the equivalent boolean expression for the following logic circuit:



Sol

(P. Q’) + (P. R’)

18. State and prove DeMorgan’s laws in boolean algebra.
Sol
DeMorgan’s Laws

The two DeMorgan’s throrems are:

(i) $\overline{XY} = \overline{X} + \overline{Y}$

This theorem states, that the complement of a product is equal to sum of complements, i.e.

complement of two or more variables used in AND gate is the same as the OR gate of the complement of each individual variables.

Truth Table

X	Y	$\overline{X}$	$\overline{Y}$	XY	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

$\overline{XY} = \overline{X} + \overline{Y}$. Hence approved

(ii) $\overline{X + Y} = \overline{X} . \overline{Y}$

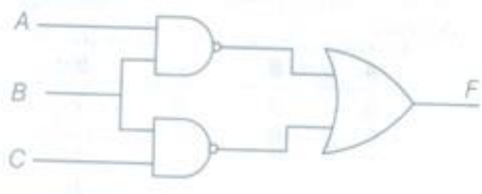
This theorem states, that the complement of sum of equal to product of complements, i.e. complement of two or more variables used in OR gate is the same as the AND gate of the complements of each individual variables.

Truth Table

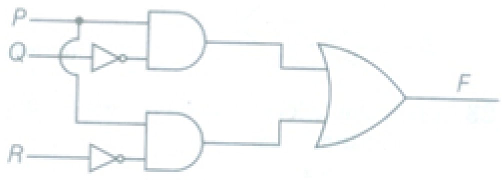
X	Y	X + Y	$\overline{X}$	$\overline{Y}$	$\overline{X + Y}$	$\overline{X}.\overline{Y}$
0	0	0	1	1	1	1
0	1	1	1	0	0	0
1	0	1	0	1	0	0
1	1	1	0	0	0	0

$\overline{X + Y} = \overline{X} . \overline{Y}$. Hence proved.

19. Write the equivalent boolean expression for the following logic circuit:



20. Write the equivalent boolean expression for the following logic circuit:



21. Verify the following algebraically:

X'.Y + X . Y' = (X' + Y') . (X + Y)

Sol X'.Y + X.Y' = (X' + Y') . (X + Y)

Taking RHS

(X' + Y') . (X + Y)

= X' X + X' Y + Y' X + Y' Y

= 0 + X' Y + Y' X + 0

[X.X' = 0]

= X'Y + X . Y'

[X.Y' = Y'.X]

= RHS

Hence proved

22. Verify the following algebraically:

(A' + B') . (A + B) = A'.B + A . B'

Sol

(A' + B') .(A + B) = A'B + AB'

LHS (A' + B') .(A + B)

=A'.A + A'B + B'A + B'B

= 0 + A'B + B'A + 0 [as A'.A = 0]

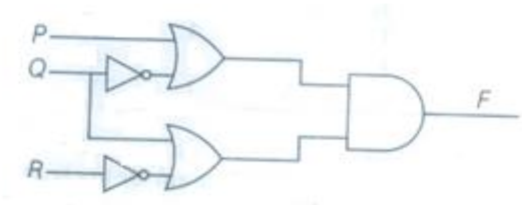
= A'B + AB' [as A.B' = B'.A]

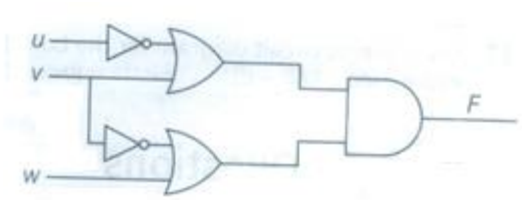
= RHS

Hence proved

23. Write the equivalent boolean expression for the following logic circuit:

24. Write the equivalent boolean expression for the following logic circuit:





25. Verify X'.Y + X . Y' + X'.Y' = (X' + Y') using truth table.

Sol

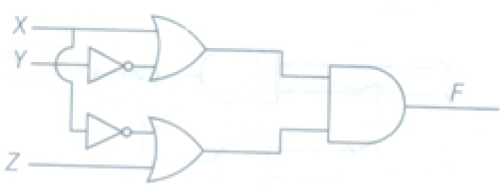
X	Y	X'	Y'	X'Y	XY'	X'Y'	X'Y + XY' + X'Y'	X' + Y'
0	0	1	1	0	0	1	1	1
0	1	1	0	1	0	0	1	1
1	0	0	1	0	1	0	1	1
1	1	1	0	0	0	0	0	0

As columns X'Y +XY' +X 'Y' + X' + Y' and X' + Y' are equal.

So, X'Y +XY' +X' Y' = X' + Y'

Hence proved

26. Write the equivalent boolean expression for the following logic circuit:



27. Represent (P + Q'. R) in POS form.

Sol

P	Q	R	Q'	Q'R	P + Q'R	Maxterm
0	0	0	1	0	0	P + Q + R
0	0	1	1	1	1	P + Q + R'
0	1	0	0	0	0	P + Q'+ R
0	1	1	0	0	0	P + Q' + R'
1	0	0	1	0	1	P' + Q + R
1	0	1	1	1	1	P + Q + R'

1	1	0	0	0	1	P + Q’+ R
1	1	1	0	0	1	P + Q’ + R’

The POS form of P + Q’R will be (P + Q + R). (P + Q’ + R). (P + Q’ + R’)

28. State the verify absorption law in boolean algebra.
Sol
Absorption law states

(i) X + XY = X

(ii) X (X + Y) = X

Truth table for X + XY = X

X	**Y**	**XY**	**X + XY**
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Hence, columns 1 and 4 are equal.

As X + XY = X

Truth table for X (X + Y) = X

X	**Y**	**X + Y**	**X(X + Y)**
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Hence, columns 1 and 4 are equal.

As X (X + Y) = X

29. Convert the following boolean expression into its equivalent canonical Sum of Product (SOP) form:

(X’+Y + Z’).(X’+Y + Z).(X’+Y’+Z).(X’+Y’+Z’)

Sol

F = (X’ + Y + Z’) .(X’ + Y + Z) .(X’ + Y’ + Z) .(X’ + Y’ + Z’)

= (10 1). (100). (110). (111)

= $M_5 . M_4 . M_6 . M_7$

= ∏ (4, 5, 6, 7)

Thus, equivalent SOP expression will be (incorporating the missing terms from POS expression)

= ∑ (0, 1, 2, 3)

i.e. $m_0 + m_1 + m_2 + m_3 = (\overline{X}. \overline{Y}. \overline{Z}) + (\overline{X}. \overline{Y}. Z) + (\overline{X}. Y. \overline{Z}) + (\overline{X}. Y. Z)$

30. Convert the following boolean expression into the equivalent canonical product of Sum (POS) form.

A. B' .C + A' .B.C + A'.B.C'

Sol

A.B'.C + A'.B. C + A'.B C'

i.e. (1 0 1) (0 1 1) (0 1 0)

$= m_5 + m_3 + m_2 = \sum (2, 3, 5)$

⟹ POS is equal to

$= \Pi (0, 1, 4, 6, 7)$

$= M_0 . M_1 . M_4 . M_6 . M_7$

$= (A + B + C). (A + B + \overline{C}) . (\overline{A} + B + C). (\overline{A} + \overline{B} + C) .(\overline{A} + \overline{B} + \overline{C}).$

31. State and verify distributive law in boolean algebra.

Sol

Distributive law states

(i) X (Y + Z) = XY + XZ and (ii) X + YZ = (X + Y) (X + Z)

Truth table for X (Y + Z) = XY + XZ

X	Y	Z	Y + Z	X.(Y + Z)	XY	XZ	XY + XZ
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

As columns X .(Y + Z) and XY + XZ are equal.

As X .(Y + Z) = XY + XZ **Hence proved**

Truth table for X + YZ) = (X + Y) + (X + Z)

X	Y	Z	YZ	X + YZ	X + Y	X + Z	(X + Y)(X + Z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0

0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

As columns (X + YZ) and (X + Y) + (X + Z) are equal.

As X + YZ = (X + Y) (X + Z)

Hence proved

32. Convert the following boolean expression into its equivalent canonical Sum of Product (SOP):

(u’ + v + w’).(u + v’ + w’).(u + v + w)

33. Given the following truth table, write the sum of products from of the function F (x, y, z):

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Sol

X’ y’ z + x’ y z’ + x y’ z’ + xyz

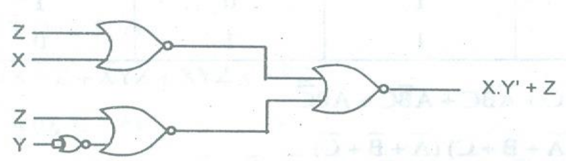
34. Prove the algebraically x’ y ‘ z’ + x’ y’ z + x’ yz’ + x’yz’ + xy’z’ + xy’z = x’ + y’.

Sol

x’ y ‘ z’ + x’ y’ z + x’ yz’ + x’yz’ + xy’z’ + xy’z = x’ + y’

L.H.S. *= x’y’ (z’ + z) + x’y (z + z’) +xy’ (z’ + z)*

= x’y’ + x’y + xy’



(z’ + z = 1, z + z’ = 1)

= x’ (y’ + y) + xy’

= x’ + xy’

35. Convert X + Y to minterms

Sol

X + Y = X . 1 + Y . 1

$= X . (Y + Y) + Y(X + X)$ (X + X = 1 complementarity law)

$= XY + XY + XY + XY$

$= XY + XY + XY + XY$

$= XY + XY + XY$ (XY + XY + XY *Idempotent* law)

36. Convert the following three input F denoted by the expression $F = \sum (0, 1, 2, 5)$ into its canonical Sum-of-Products form.

Sol

If three inputs we take as X, Y and Z then

$$F = m_0 + m_1 + m_2 + m_5$$

$$m_0 = 000 \longrightarrow \overline{X}\,\overline{Y}\,\overline{Z}$$

$$m_1 = 001 \longrightarrow \overline{X}\,\overline{Y}\,Z$$

$$m_2 = 010 \longrightarrow \overline{X}\,Y\,\overline{Z}$$

$$m_5 = 101 \longrightarrow X\,\overline{Y}\,Z$$

Canonical *S-O-P* form of the expression is

$$\overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}\,\overline{Y}\,Z + \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,Z$$

37. Simplify *ABCD + ABCD + ABCD + ABCD.*

Sol

$= AB'C(D' + D) + ABC(D' + D)$

$= ABC . 1 + ABC . 1$ (D + D′= 1)

$= AC(B + B)$

$= AC . 1 = AC$ (B + B′ = 1)

38. Provide that $X. (X + Y) = X$ by algebraic method.

Sol

$L.H.S = X . (X + Y)$

$= X . X + X . Y$

$= X + X .Y$ (X . X = X)

$= X . (1 + Y)$

$= X . 1 = X = R.H.S$ (1 + Y = 1)

$= X . 1 = X = R.H.S$

39. Verify $X . Y' + Y' .Z = X. Y' .Z' + X .Y' .Z' + X' .Y' .Z$ algebraically.

Sol

LHS = $X .Y' + Y' . Z$

$= X . Y' (Z + Z') + (X + X') Y' . Z$ (Z + Z′ = 1 *and* X + X′ = 1)

$= XY' Z + XY'Z + XY'Z + X'Y'Z$

$= XY'Z + XY'Z + X'Y'Z$

= RHS, Hence Proved.

40. Perform the following:

(a) State and prove the De Morgan's Theorem (Any One) Algebraically.

Sol

(a) DeMorgan's Theorem's state that

(i) $\overline{X + Y} = \overline{X} . \overline{Y}$ (ii) $\overline{X . Y} = \overline{X} + \overline{Y}$

Proof. Assuming that DeMorgan's laws are true. That means, all Boolean laws should hold on it. Let

X + Y = P

As from given theorem (i), we get

$\overline{Q} = \overline{X} . \overline{Y}$

Since Boolean laws on hold on it, complementarity law should also hold on it.

⟹ $P + \overline{P} = 1$ and $P . \overline{P} = 0$

P + P = 1

Replacing value of P, we get

$P + \overline{X} . \overline{Y} + 1$

LHS = (P + X) (P + Y)

Replacing value of P, we get

$(X + Y + \overline{X})(X + Y + \overline{Y}) = (X + \overline{X} + Y)(X + 1)$

= (1 + Y) (X + 1) (X + X'=y+ Y' = 1)

= 1 . 1 (1 + Y = 1 + X = 1)

= 1 = RHS

Similarly, replacing P and $\overline{P}$ in $P . \overline{P} = 0$, we get

$(X + Y)\, \overline{X} . \overline{Y} = 0$

$LHS = X\overline{XY} + Y\overline{X}Y$

= 0 + 0 (XX' = 0, YY' = 0)

= 0 = RHS.

Thus, complementarity law fully holds on it.

⟹ DeMorgan theorem is a legal Boolean algebra theorem.

41. State and prove the absorption algebraically.

Sol

Absorption law states that

(i) X + XY = X and (ii) X (X + Y) = X

Proof. (i) X + XY = X

LHS + XY = X (1 + Y)

= X . 1 [1 + Y = 1]

= X = RHS Hence Proved.

42. Given the following truth table, derive a Sum of Product (SOP) and Product of Sum (POS) form
of Boolean expression from it:

X	Y	Z	G (X, Y ,Z)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	0	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

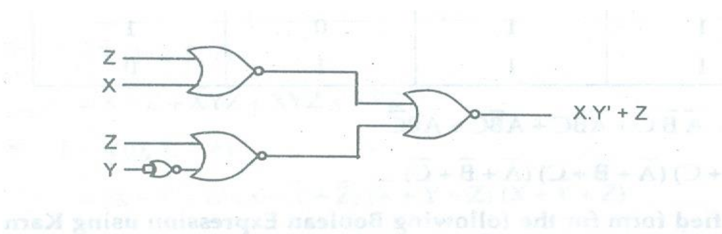
43. Given the following truth table, derive a sum of product (SOP) and Product of Sum (POS) form of Boolean expression from it.

A	*B*	*C*	*F (A, B ,C)*
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

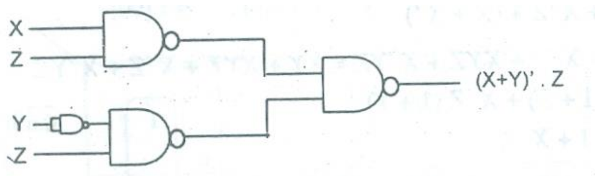
44. Represent the Boolean expression X. Y' + Z with the help of NOR gates only.
Sol
X . Y' + Z

= Z + XY' [X + Y = Y + X]

= (Z + X) (Z + Y') [X + YZ = (X + Y) (X + Z)]



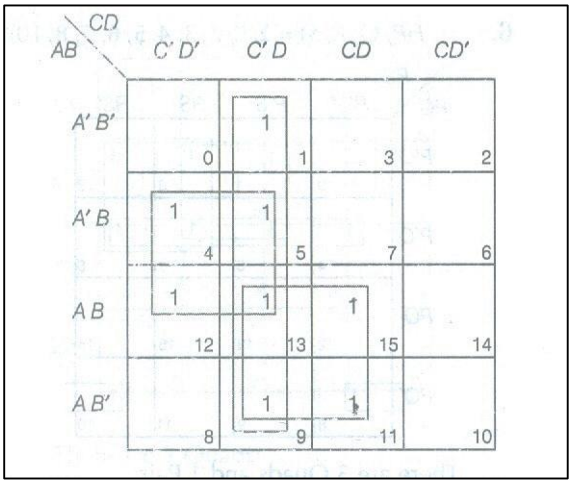
45. Represent the Boolean expression (X + Y') .Z with the help of NAND gates only.
Sol
(X + Y') . Z =X . Z + Y' . Z



3 Marks Questions

1. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
   F(A, B, C, D) = ∑(1, 4, 5, 9, 11, 12, 13, 15)

   Sol



There are 3 Quads:

Quad 1 ($m_1 + m_5 + m_9 + m_{13}$) reduces to *C'D*

Quad 2 ($m_4 + m_5 + m_{12} + m_{13}$) reduces to *BC'*

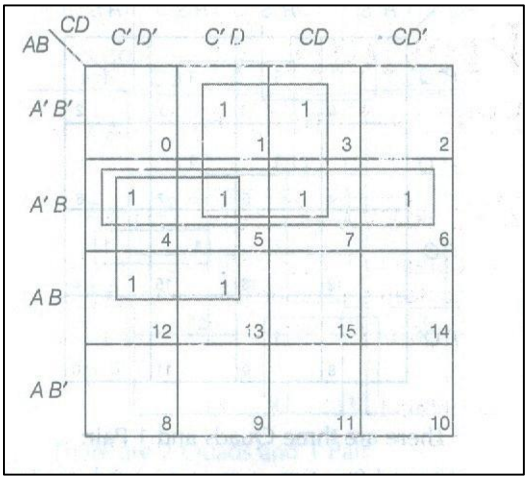
Quad 3 ($m_9 + m_{11} + m_{13} + m_{15}$) reduces to *AD*

Hence, the final expression is:

F(A, B, C) = C'D + BC' + AD

2. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
   F(A, B, C, D) = ∑(1, 3, 4, 5, 6, 7, 12, 13)

   Sol



There are 3 Quads:

Quad 1 ($m_1 + m_3 + m_5 + m_7$) reduces to *A’D*

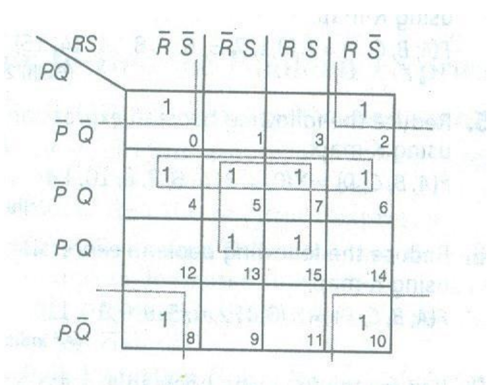
Quad 2 ($m_4 + m_5 + m_6 + m_7$) reduces to *A’B*

Quad 3 ($m_4 + m_5 + m_{12} + m_{13}$) reduces to *BC’*

Hence, the final expression is:

F(A, B, C,D) = A’D + A’B + B C’

3. Obtain a simplified form for the following boolean expression using Karnaugh’s Map:
   F(P, Q, R, S) = ∑(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)
   Sol



There are three Quads:

Quad 1 ($m_4 + m_5 + m_6 + m_7$) reduces to $\overline{P}Q$

Quad 2 ($m_5 + m_7 + m_{13} + m_{15}$) reduces to *QS*

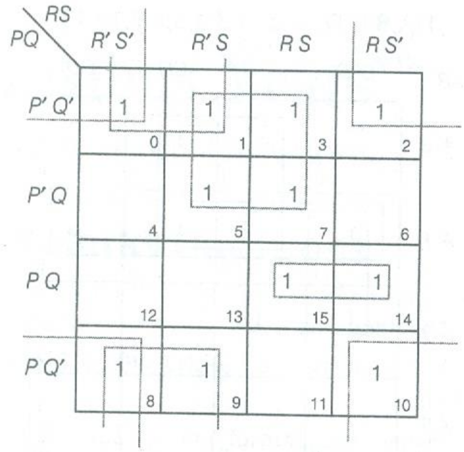
Quad 3 ($m_0 + m_2 + m_8 + m_{10}$) reduces to $\overline{QS}$

Hence, the final expression is:

$F(P, Q, R, S) = \overline{P}Q + QS + \overline{QS}$

4. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
   H(P, Q, R, S) = ∑(0, 1, 2, 3, 5, 7, 8, 9, 10, 14, 15)

Sol



There are three Quads and 1 Pair:

Quad 1 ($m_0 + m_2 + m_8 + m_{10}$) reduces to $\overline{QS}$

Quad 2 ($m_1 + m_3 + m_5 + m_7$) reduces to $\overline{P}S$

Quad 3 ($m_0 + m_1 + m_8 + m_9$) reduces to $\overline{QR}$

Pair 1 ($m_{14} + m_{15}$) reduces to PQR

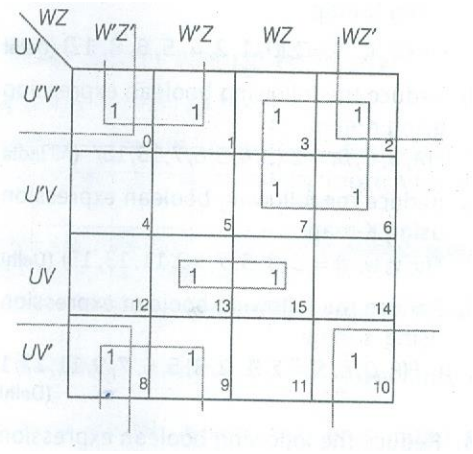
Hence, the final expression is:

H(P,Q,R,S) = Q' S' + P' S + Q' R' + *PQR*

5. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
   F(U, V, W, Z) = ∑(0, 1, 2, 3, 6, 7, 8, 9, 10, 13, 15)

Sol

0. *F (U,V, W, Z)* = **∑** (0, 1, 2, 3, 6, 7, 8, 9, 10, 13, 15)



There are three Quads and 1 Pair:

Quad 1 ($m_0 + m_2 + m_8 + m_{10}$) reduces to $\overline{V}\,\overline{Z}$

Quad 2 ($m_0 + m_1 + m_8 + m_9$) reduces to $\overline{V}\,\overline{W}$

Quad 3 ($m_2 + m_3 + m_6 + m_7$) reduces to $\overline{U}$ W

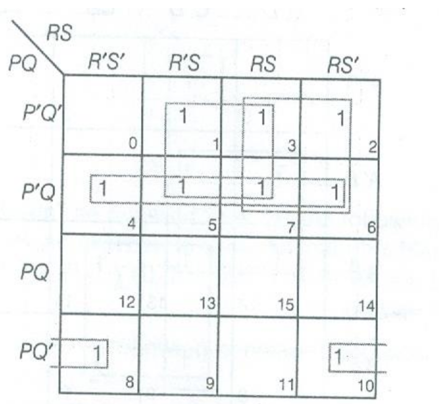
Pair 1 ($m_{13} + m_{15}$) reduces to *UVZ*

Hence, the final expression is:

F(U,V,W,Z) = V' Z' + V' W' + U' W +UVZ

6. Reduce the following boolean expression using K-map:
   F(P, Q, R, S) = ∑(1, 2, 3, 4, 5, 6, 7, 8, 10)

   Sol



There are three Quads and 1 Pair:

Quad 1 ($m_1 + m_3 + m_5 + m_7$) reduces to *P'S*

Quad 2 ($m_4 + m_5 + m_6 + m_7$) reduces to *P'Q*

Quad 3 ($m_2 + m_3 + m_6 + m_7$) reduces to *P'R*

Pair 1 ($m_8 + m_{10}$) reduces to PQ'S'

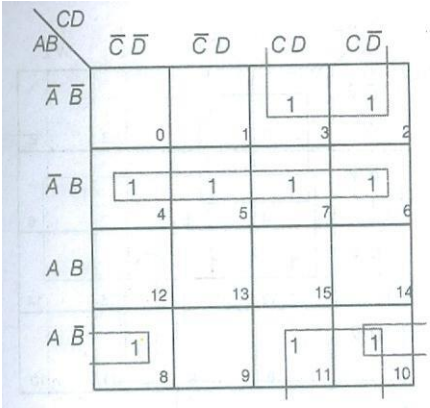
Hence, the final expression is:

F(P, Q, R, S) = P'S + P'Q + P'R + PQ'S'

7. Reduce the following boolean expression using K-map:

F(A, B, C, D) = ∑(2, 3, 4, 5, 6, 7, 8, 10, 11)

Sol



There are 2 Quads and 1 Pair:

Quad 1 ($m_4 + m_5 + m_6 + m_7$) reduces to $\overline{A}B$

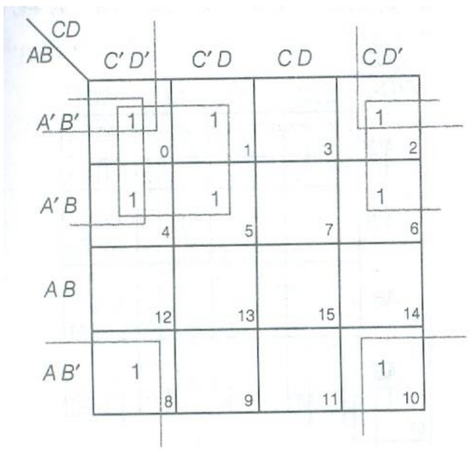
Quad 2 ($m_2 + m_3 + m_{10} + m_{11}$) reduces to $\overline{B}\,C$

Pair 1 ($m_8 + m_{10}$) reduces to $A\overline{BD}$

Hence, the final expression is:

F(A, B, C,D) = $\overline{A}B + \overline{B}C + A\overline{BD}$

8. Reduce the following boolean expression using K-map:
F(A, B, C, D) = ∑(0, 1, 2, 4, 5, 6, 8, 10,)
Sol



There are 3 Quads:

Quad 1 ($m_0 + m_1 + m_4 + m_5$) reduces to *A′C′*

Quad 2 ($m_0 + m_2 + m_8 + m_{10}$) reduces to B′ D′

Quad 3 ($m_0 + m_2 + m_4 + m_6$) reduces to *A′ D′*

Hence, the final expression is:

F(A, B, C,D) = A′C′ +B′ D′ + A′ D′

9. Reduce the following boolean expression using K-map:
   F(P, Q, R, S) = ∑(0, 1, 2, 4, 5, 6, 8, 12)

10. Reduce the following boolean expression using K-map:
    F(A, B, C, D) = ∑(3, 4, 5, 6, 7, 13, 15)

11. Reduce the following boolean expression using K-map:
    F(u, v, w, z) = ∑(3, 5, 7, 10, 11, 13, 15)

12. Reduce the following boolean expression using K-map:
    F(P, Q, R, S) = ∑(1, 2, 3, 5, 6, 7, 9, 11, 12, 13, 15)

13. Reduce the following boolean expression using K-map:
    H(u, v, w, z) = ∑(0, 1, 4, 5, 6, 7, 11, 12, 13, 14, 15)

14. Reduce the following boolean expression using K-map:
    F(A, B, C, D) = ∑(0, 1, 2, 3, 6, 7, 8, 11, 14, 15)

15. Reduce the following boolean expression using K-map:
    F(A, B, C, D) = ∑(0,2, 3, 4, 6, 7, 8, 10, 12,)

16. Reduce the following boolean expression using K-map:
    F(A, B, C, D) = ∑(0, 1, 2, 4, 5, 8, 9, 10, 11,)

17. Reduce the following boolean expression using K-map:
    F(A, B, C, D) = ∑(1, 3, 4, 5, 7, 9, 11, 12, 13, 14)

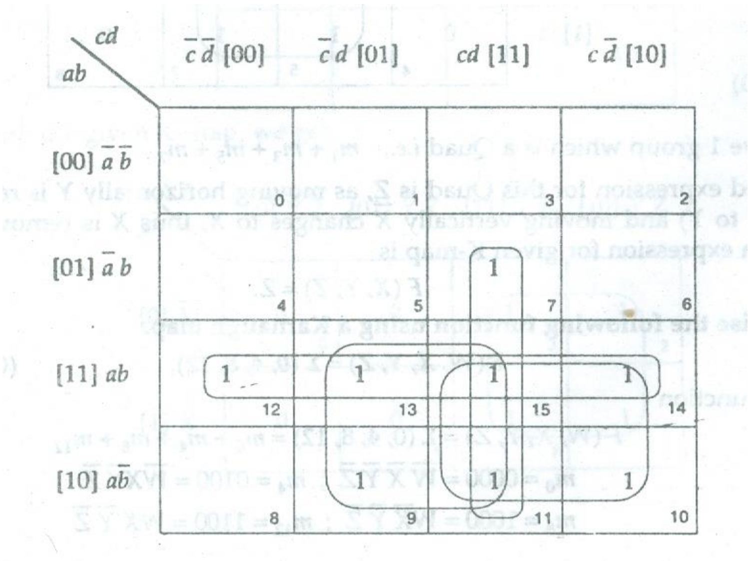
18. If *F* (a, b, c, d) = ∑ (0, 2, 4, 5, 7, 8, 10, 12, 13, 15), obtain the simplified form using K-Map.

19. What is the simplified Boolean equation for the following K-map.

ab \ cd	$\bar{c}\bar{d}$ [00]	$\bar{c}d$ [01]	cd [11]	$c\bar{d}$ [10]
[00] $\bar{a}\bar{b}$	0	1	3	2
[01] $\bar{a}b$	4	5	1 7	6
[11] ab	1 12	1 13	1 15	1 14
[10] $a\bar{b}$	8	1 9	1 11	1 10

Sol

Completing the Karnaugh map by entering 0's in the empty squares with their minterm's subscripts and then by encircling all possible groups, we get the following K-map.



3 Quads and a pair is marked:

Quad 1 ($m_{12} + m_{13} + m_{14} + m_{15}$) reduces to *ab*

Quad 2 ($m_9 + m_{11} + m_{13} + m_{15}$) reduces to *ad*

Quad 3 ($m_{10} + m_{11} + m_{14} + m_{15}$) reduces to *ac*

Pair 1 ($m_2 + m_{10}$) reduces to b c d.

Hence the final reduced expression is

$$F = ab + ad + ac + bcd$$

20. Minimise the following function using a Karnaugh map.

$$F(W, X, Y, Z) = \sum (0, 4, 8, 12).$$

Sol

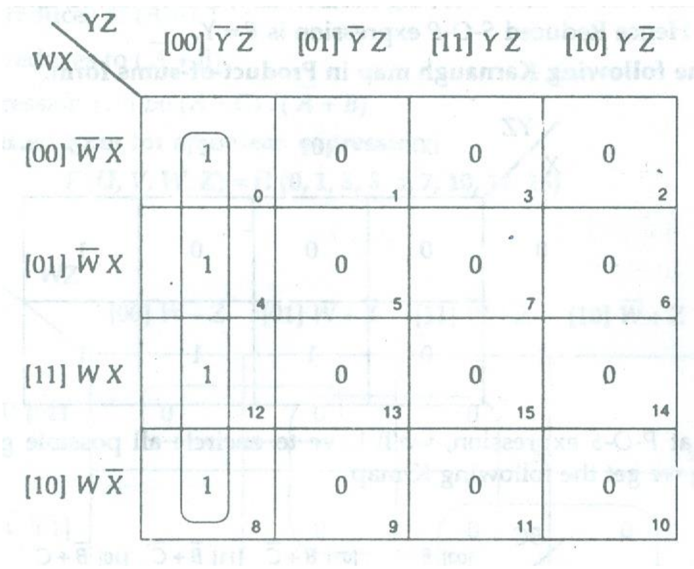
1. Given function

$$F(W, X, Y, Z) = \sum (0, 4, 8, 12) = m_0 + m_4 + m_8 + m_{12}$$

$$m_0 = 0000 = W X Y Z;\ m_4 = 0100 = W X Y Z$$

$$m_0 = 1000 = W X Y Z;\ m_{12} = 1100 = W X Y Z$$

Mapping the given function on a K-map, we get



Only 1 group is here, a Quad ($m_0 + m_4 + m_{12} + m_8$)

Reduced expression for this quad is $\overline{Y}\,\overline{Z}$ thus, final reduced expression is $F = \overline{Y}\,\overline{Z}$.

21. Obtain the simplified form a Boolean expression:

$F(u, v, w, z) = \sum(0, 1, 3, 5, 7, 9, 10, 11, 12, 13, 14, 15)$

using Karnaugh Map.

Sol

2. There are 4 gps: 1 octet, 2 Quads and 1 pair

Octet ($m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$) reduces to z.

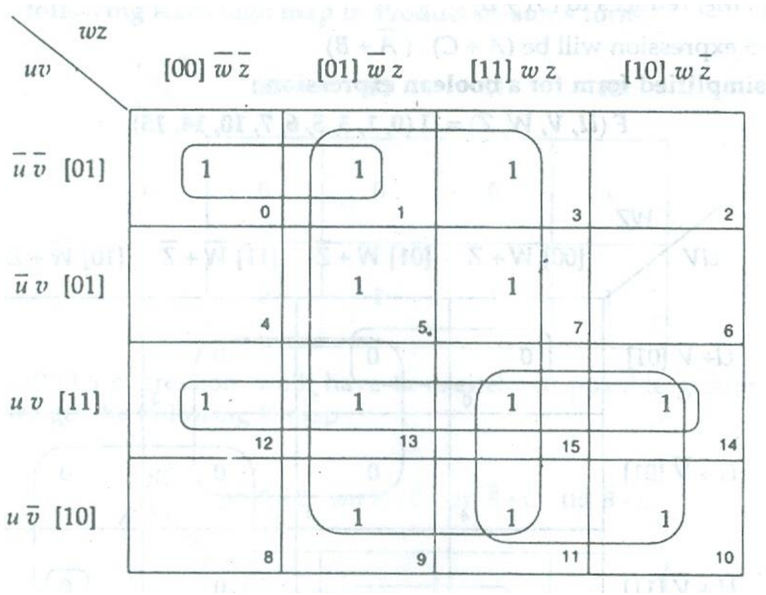
Quad 1 ($m_{12} + m_{13} + m_{14} + m_{15}$) reduces to uv.

Quad 2 ($m_{10} + m_{11} + m_{14} + m_{15}$) reduces to uw.

Pair ($m_0 + m_1$) reduces to $\overline{u}\,\overline{v}\,\overline{w}$

The final reduced expression is

$u\,v\,w + uv + uw + z$



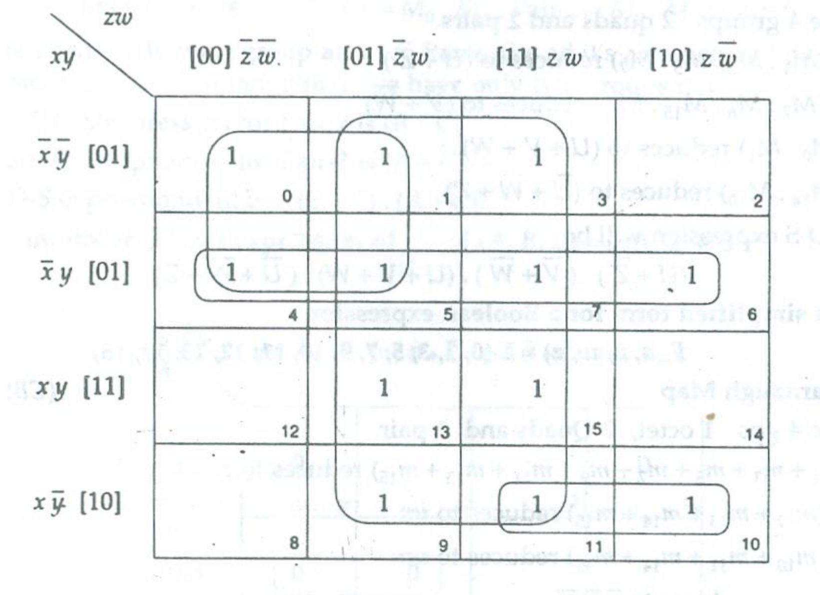
22. Obtain a simplified form for a Boolean expression

$F(x, y, z, w) = \sum (0, 1, 3, 4, 5, 6, 7, 9, 10, 11, 13, 15)$

using Karnaugh Map.

Sol

3.



There are 4 groups: 1 octet, 2 Quads and 1 pair.
The octet ($m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$) reduces to w.
The quad 1 ($m_0 + m_1 + m_4 + m_5$) reduces to $\bar{x}\,\bar{z}$
The quad 2 ($m_4 + m_5 + m_6 + m_7$) reduces to $\bar{x}\,y$.
The pair ($m_0 + m_{11}$) reduces to $x\,\bar{y}\,z$
The final reduced expression is

F (x, y, z, w) $w + \bar{x}\,\bar{z} + \bar{x}\,y + x\,\bar{y}\,z$